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Substituting in (6), we have

$$\int x^m \log^n x dx = \frac{x^{m+1}}{m+1}\left[\log^n x - \frac{n \log^{n-1} x}{m+1} + \frac{n(n-1)\log^{n-2} x}{(m+1)^2} \dots \right.$$
$$\left. (-1)^n \frac{n(n-1)\dots(3)(2)(1)}{(m+1)^n}\right]. \quad (7)$$

If in (7) $m = 0$, and we take the integral between the limits $x = 0$ and $x = 1$, we have

$$\int_0^1 \log^n x dx = (-1)^n n(n-1)(n-2)\dots 3 \,.\, 2 \,.\, 1.$$

Ex. 2. $\int a^x x^n dx = ?$

Make $u = a^x \div (\log a)$, and $v = x$. Hence,

$$\int udv = \frac{a^x}{\log^2 a},$$

$$\int^2 udv^2 = \frac{a^x}{\log^3 a},$$

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$$\int^n udv^n = \frac{a^x}{\log^{n+1} a}.$$

Substituting in (6),

$$\int a^x x^n dx = \frac{a^x}{\log a}\left[x^n - \frac{nx^{n-1}}{\log a} + \frac{n(n-1)x^{n-2}}{\log^2 a} \dots (-1)^n \frac{n(n-1)\dots 3.2.1}{\log^n a}\right].$$

In a similar manner we may integrate a number of other important differentials, such as

$$\sin^m x \cos^n x dx, \qquad (a+bx^n)^p x^m dx, \text{ \&c.}$$

SOLUTION OF PROB. 334, BY W. E HEAL.—(See pp. 60, 98, Vol. VIII)

Let the given ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

The locus of intersection of tangents which intersect at a constant angle, φ, is

$$[(x^2+y^2)-(a^2+b^2]^2 \tan^2\varphi = 4(b^2x^2+a^2y^2-a^2b^2). \quad (A)$$

(Salmon's Conics, 5th edition, page 161.)

The polar of every point on this curve touches the required curve and conversely the polar of every point on the required curve touches the curve (A).

Let $(\varkappa, \lambda)$ be the coordinates of any point on the required curve.

The polar of this point is

$$\frac{\varkappa x}{a^2} + \frac{\lambda y}{b^2} = 1. \quad (B)$$

Making the equations homogeneous by writing in them $z = 1$,

$$[(x^2+y^2)-(a^2+b^2)z^2]^2\tan^2\varphi = 4z^2(b^2x^2+a^2y-a^2b^2z^2). \qquad (C)$$

$$\frac{\varkappa x}{a^2}+\frac{\lambda y}{b^2} = z. \qquad (D)$$

Eliminating z, the coordinates of the points in which (C) and (D) intersect are given by the equation

$$\left[(x^2+y^2)-(a^2+b^2)\left(\frac{\varkappa x}{a^2}+\frac{\lambda y}{b^2}\right)^2\right]^2\tan^2\varphi = 4\left(\frac{\varkappa x}{a^2}+\frac{\lambda y}{b^2}\right)^2$$
$$\times\left[b^2x^2+a^2y-a^2b^2\left(\frac{\varkappa x}{a^2}+\frac{\lambda y}{b^2}\right)^2\right].$$

Expanding and reducing, we get

$$Ax^4+Bx^3y+Cx^2y^2+Dxy^3+Ey^4 = 0. \qquad (E)$$

Where

$$A = b^4[a^2-b^2(a^2+b^2)\varkappa^2]^2\tan^2\varphi-4a^2b^8\varkappa^2+4a^2b^{10}\varkappa^4,$$

$$B = 4[a^2b^6(a^2+b^2)^2\varkappa^3\lambda-a^4b^4(a^2+b^2)\varkappa\lambda]\tan^2\varphi-8a^4b^6\varkappa\lambda+16a^4b^8\varkappa^3\lambda,$$

$$C = [2a^4b^4-2a^2b^2(a^2+b^2)(a^4\lambda^2+b^4\varkappa^2)+6a^4b^4(a^2+b^2)^2\varkappa^2\lambda^2]\tan^2\varphi$$
$$-4(a^6b^4\lambda^2+a^4b^6\varkappa^2)+24a^6b^6\varkappa^2\lambda^2,$$

$$D = 4[a^6b^2(a^2+b^2)^2\varkappa\lambda^3-a^4b^4(a^2+b^2)\varkappa\lambda]\tan^2\varphi-8a^6b^4+16a^8b^4\varkappa\lambda^2,$$

$$E = a^4[b^2-a^2(a^2+b^2)\lambda^2]^2\tan^2\varphi-4a^8b^2\lambda^2+4a^{10}b^2\lambda^4.$$

But if (D) is tang't to (C), equation (E) must have equal roots and therefore its discriminant vanishes. Therefore

$$4(12AE-3BD+C^2)^3 = (72ACE+9BCD-27AD^2-27EB^2-2C^3)^2.$$

(Salmons Higher Algebra, 3rd edition, p. 306.)

This is an equation between the coordinates $\varkappa$, λ and is, therefore the required envelope.

NOTE ON AN INDETERMINATE EQN., BY WM. HOOVER, A. M.—The following quotation is from a communication by M. E. Catalan to *Journal de Mathematiques* for August, 1882.

"The identity

$$(a+b)^2(a-2b)^2(b-2a)^2+27a^2b^2 = 4(a^2-ab+b^2)^3,$$

easy of verification, gives an indefinite number of solutions, in entire numbers, of

$$x^2+3y^2 = z^3.$$

"We can take

$x = \frac{1}{2}(a+b)(a-2b)(b-2a$, $y = \frac{3}{2}ab(a-b)$, and $z = a^2-ab+b^2$."